

ON POSSIBLE MODIFICATIONS OF GRAVITATION IN THE (SUB)MILLIMETER RANGE

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I discuss possible modifications of gravitation at short distances in string theories with large internal dimensions and low string scale. The modifications are due to the change of Newton's law in the presence of (sub)millimeter-size transverse dimensions, or due to the existence of light scalars with masses as small as 10^{-3} eV, related to the mechanism of supersymmetry breaking, mediating new Yukawa-type forces with strength comparable to gravity. These modifications are testable in "tabletop" experiments that measure gravity at such short distances. (Invited talk given at the XXXIV Rencontres de Moriond on *Gravitational Waves and Experimental Gravity*, Les Arcs, France, January 23-30, 1999)

1 TeV^{-1} dimensions, supersymmetry breaking and submm forces

String theories provide at present the only consistent framework of quantum gravity and unification of all fundamental interactions. Traditionally, it was believed that they become relevant only at Planckian energies due to the simple tree-level expression for the string scale,

$$M_H \simeq g M_P, \quad (1)$$

valid in the context of perturbative heterotic theory. Here, $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass, and $g \simeq 1/5$ is the gauge coupling at the heterotic string (unification) scale $M_H \simeq 10^{18}$ GeV. Despite this relation, there are however physical motivations which indicate that large volume compactifications may be relevant for physics. One of them comes from the problem of supersymmetry breaking by the process of compactification which relates the breaking scale to the size of some internal compact dimension(s). The latter should therefore be in the TeV^{-1} region in order to keep the scale of supersymmetry breaking close to electroweak energies¹.

An immediate defect of this scenario is that the ten-dimensional (10d) string coupling is huge invalidating the perturbative description. From the 4d point of view, the problem arises due to the infinite massive tower of Kaluza-Klein (KK) excitations that are produced at energies above the compactification scale and contribute to physical processes:

$$M^2 = M_0^2 + \frac{n^2}{R^2}; \quad n = 0, \pm 1, \dots, \quad (2)$$

where M_0 is a 5d mass and R is the radius of an extra dimension. If the theory makes sense for some special models, a possible way out consists of imposing a set of conditions to the low energy theory that prevent the effective couplings to diverge. An example of such conditions is that KK modes should be organized into multiplets of $N = 4$ supersymmetry, containing for each spin-1 4 Weyl-fermions and 6 scalars with the same quantum numbers, so that their contribution to beta-functions vanish for every $n \neq 0$ and gauge couplings remain finite.

The breaking of supersymmetry by compactification is realized through boundary conditions and is similar to the effects of finite temperature with the identification $T \equiv R^{-1}$. It follows that the breaking is extremely soft and insensitive to the ultraviolet (UV) physics above

the compactification scale. The summation over the KK excitations amounts to inserting the Boltzmann factors $e^{-E/T}$ to all thermodynamic quantities –or equivalently to the soft breaking terms– that suppress exponentially their UV behavior^{1,2}. This is in contrast to the behavior of supersymmetric couplings that generally blow up, unless special conditions are imposed.

The extreme softness of supersymmetry breaking by compactification has two important phenomenological consequences: (i) a particular spectroscopy of superparticles that differs drastically from other scenarios^{1,3}, and (ii) a new scalar force in the (sub)mm range mediated by the scalar modulus whose vacuum expectation value (VEV) determines the radius R of the TeV dimension used to break supersymmetry². One can easily check by direct dimensional reduction of the higher-dimensional Einstein action that the scalar field associated to the radius modulus with canonical kinetic terms is $\phi = \ln R$ in Planck units. Its VEV corresponds to a flat direction of the scalar potential, which is lifted only after supersymmetry breaking, through a potential generated by radiative corrections. The softness of the breaking mechanism implies that the vacuum energy V behaves for large R as

$$V \sim T^4 \equiv \frac{1}{R^4}, \quad (3)$$

up to logarithmic higher loop corrections. Notice the absence of a “quadratically divergent” $M_P^2 R^{-2}$ contribution. It follows that the mass of the radius modulus ϕ is of order^{4,2}

$$m_\phi \sim R^{-2}/M_P \simeq 10^{-4} - 10^{-2} \text{ eV}, \quad (4)$$

for a compactification scale $R^{-1} \simeq 1 - 10 \text{ TeV}$. This corresponds to Compton wavelengths of the modulus in the range of 1 mm to 10 μm .

The coupling of this light scalar to matter arises dominantly through the dependence on ϕ of the QCD scale Λ_{QCD} , or equivalently of the QCD gauge coupling α_{QCD} . The coupling of the radius modulus to nucleons is given by the derivative of the nucleon mass relative to ϕ , $\frac{\partial m_N}{\partial \phi}$. Since the graviton coupling to nucleons is m_N/M_P , the coupling α_ϕ of the modulus relative to gravity is

$$\alpha_\phi = \frac{1}{m_N} \frac{\partial m_N}{\partial \phi} = \frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \ln R} = -\frac{1}{b_{\text{QCD}}} \frac{\partial}{\partial \ln R} \frac{2\pi}{\alpha_{\text{QCD}}}, \quad (5)$$

with b_{QCD} the one-loop QCD beta-function coefficient. Since in these models, in the absence of supersymmetry breaking all couplings of the effective field theory are well behaved in the large radius limit, while after supersymmetry breaking all soft masses are proportional to $1/R$, one has

$$\frac{\partial}{\partial \ln R} \frac{2\pi}{\alpha_{\text{QCD}}} = b_{\text{SQCD}} - b_{\text{QCD}}, \quad (6)$$

with b_{SQCD} the supersymmetric beta-function. Using $b_{\text{SQCD}} = 3$ and $b_{\text{QCD}} = 7$, one obtains $\alpha_\phi = 1 - b_{\text{SQCD}}/b_{\text{QCD}} = 4/7$. Thus, the force between two pieces of matter mediated by the radius modulus is $(4/7)^2 \simeq 1/3$ times the force of gravity².

In principle there can be other light moduli which couple with larger strengths. For example the dilaton, whose VEV determines the (logarithm of the) string coupling constant, if it does not acquire large mass from some dynamical supersymmetric mechanism, leads to the strongest effect. Its coupling is $\alpha_{\text{dilaton}} = \ln(M_P/\Lambda_{\text{QCD}}) \sim 44$ ⁵, which corresponds to a strength ~ 2000 times bigger than gravity.

In fig. 1 we depict the theoretical predictions together with information from previous, present and upcoming experiments. The vertical axis is the strength, α^2 , of the force relative to gravity; the horizontal axis is the Compton wavelength of the exchanged particle; the upper scale shows the corresponding value of the large radius in TeV. The solid lines indicate the present limits from the experiments indicated⁶. The excluded regions lie above these solid lines. Our

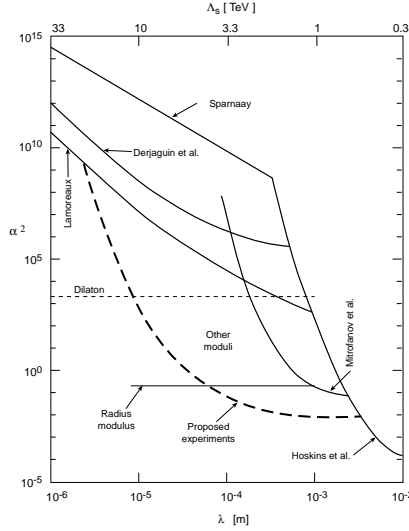


Figure 1: Strength of the modulus force relative to gravity (α^2) versus its Compton wavelength (λ).

theoretical prediction for the radius modulus is the thin horizontal line at $1/3$ times gravity. The dilaton-mediated force is the thin dashed horizontal line at ~ 2000 times gravity. Other moduli may mediate forces between the above two extremes.

Of course, measuring gravitational strength forces at such short distances is quite challenging. The most important background is the Van der Waals force. The Van der Waals and gravitational forces between two atoms are equal to each other when the atoms are about $100 \mu\text{m}$ apart. Since the Van der Waals force falls off as the 7th power of the distance, it rapidly becomes negligible compared to gravity at distances exceeding $100 \mu\text{m}$. The most important part of the figure is the dashed thick line; it is the expected sensitivity of the present and upcoming experiments, which will improve the actual limits by almost 6 orders of magnitude and –at the very least– they will, for the first time, measure gravity to a precision of 1% at distances of $\sim 100 \mu\text{m}$ ⁷.

2 TeV strings, submm dimensions and signals in particle colliders

As mentioned before, the main defect of the above heterotic string models is the strong 10d coupling due to the large internal dimension(s) used to break supersymmetry. Fortunately, this problem can be addressed using the recent results on string dualities, by considering the corresponding dual weakly coupled theory. In the minimal case of one or two large dimensions, the dual weakly coupled description is provided by type II string theory, whose tension appears as a non-perturbative threshold below the heterotic scale⁸. For instance, in the case of one TeV dimension, the type II scale appears at intermediate energies of order 10^{11} GeV, and the theory above the TeV but below the intermediate scale becomes effectively six-dimensional and is described by a tensionless string⁹ with no gravity. This 6d theory possesses a non-trivial infrared fixed point which encodes the conditions needed to be imposed in the low energy couplings, in order to ensure a smooth UV behavior. These conditions generalize the requirement of $N = 4$ supersymmetry for the KK excitations, that keep the gauge couplings well behaved, to the Yukawa couplings of the theory. A generic property of these models is that chiral matter is localized at particular points of the large internal dimensions. As a result, quarks and leptons have not KK TeV excitations, which is welcome also for phenomenological reasons in order to avoid fast proton decay^{1,10}.

The main prediction of these theories in particle accelerators is the existence of KK excitations (2) for all Standard Model gauge bosons that can be produced for instance in hadron

colliders, such as the Tevatron and LHC, through Drell-Yan processes¹¹. The corresponding KK resonances are narrow with a ratio of their width to their mass $\Gamma/m \sim g^2 \sim$ a few per cent, and thus the typical expected signal is the production of a double resonance corresponding the first KK mode of the photon and Z , very nearly spaced the one from the other. The current limits on the size of large dimensions arise from the bounds of compositeness or from other indirect effects, such as in the Fermi constant and LEP2 data, and lie in the range of $1 - 2$ TeV^{10,12}. On the other hand, direct production of KK states in LHC is possible for compactification scales of less than about 5 TeV.

In the case of two TeV dimensions, the type II string scale appears also at the TeV, while its coupling is infinitesimally small $\sim 10^{-14}$ ⁸. As a result, despite the fact that the string scale of the dual theory is so low, it has no observable effects in particle accelerators and the main experimental signal continues to be the existence of KK excitations along the two TeV dimensions with gauge interactions. The predictions are thus very similar to the previous case described above. It is amazing that the main features of these models were captured already in the context of heterotic string despite its strong coupling. When there are more than two large dimensions, a new phenomenon appears. The dual weakly coupled theory, that can now be either type II or type I, contains also some extra large dimensions of size much larger than $\text{TeV}^{-1} \simeq 10^{-16}$ cm, where only gravity propagates.

It is now clear from the above discussion that in other than the heterotic perturbative descriptions of string theory, the string scale is not tied to the Planck mass by a simple relation like (1), but it corresponds to an arbitrary parameter that can be everywhere above the TeV scale¹³. In particular, if it is at the TeV, the gauge hierarchy problem would be automatically solved without the need of low energy supersymmetry or technicolor. The price to pay is to introduce an extremely small string coupling and/or, alternatively, extra large dimensions in order to account for the hierarchy M_{str}/M_P .

For instance, it was recently proposed that the fundamental scale of gravity could be at the TeV if there were such extra large dimensions where only gravity propagates, while the Standard Model should be localized on a 3d wall^{14,15}. The observed weakness of 4d gravity would then be a consequence of the higher dimensional Gauss-law:

$$G_N = \frac{G_{N(4+p)}}{R_\perp^p}, \quad (7)$$

where $G_{N(4+p)}$ is the $(4+p)$ -dimensional Newton's constant and R_\perp denotes the (common) size of p extra large spacial dimensions. This scenario can be realized naturally in the context of type I theory of closed and open strings^{15,16}. Gravity is described by closed strings, while gauge interactions are described by open strings whose ends are confined to propagate on subspaces, called D-branes. The theory is weakly coupled when the size of the internal dimensions parallel to the D-brane where the Standard Model is localized is of order of the string scale M_I^{-1} , while there is no restriction on the size of transverse dimensions. One then obtains that the type I string scale is simply related to the higher-dimensional Newton's constant

$$G_{N(4+p)} \simeq g^2 M_I^{-2-p}, \quad (8)$$

and is therefore a free parameter, while the string coupling is given by the square of the gauge coupling. This expression should be compared with the corresponding relation (1) in the heterotic string, where the string scale is fixed by the 4d Planck mass.^a

Using eqs. (7) and (8) and taking the type I string scale to be at the TeV, one finds a size for the transverse dimensions varying from 10^8 km, .1 mm, up to .1 fermi for $p = 1, 2$, or 6 large

^aNote that in the context of type II theories, the string tension is decoupled from the scale where gravity becomes strong and can be lowered independently by decreasing the string coupling which is a free parameter.

dimensions. With the exception of $p = 1$ which is obviously experimentally excluded, all other values are consistent with observations^{14,17,18}. The strongest bound applies for $p = 2$ and comes from graviton emission on cooling of supernovae which restricts the 6d Planck scale to be larger than about 50 TeV¹⁹, implying $M_I \gtrsim 7$ TeV. The main distinct experimental signals of these theories in particle accelerators are¹⁵:

- (i) Production of higher spin Regge-excitations for all Standard Model particles with the same quantum numbers and mass-squared increasing linearly with the spin. For instance, the excitations of the gluon could show up as a series of peaks in jet production at LHC. However, the corresponding resonances are very narrow, with a ratio of their width to their mass $\Gamma/m \sim g^4 \sim$ a few per thousand, and thus are difficult to detect.
- (ii) Graviton emission into the extra dimensions leading to events of jets + missing energy. LHC is sensitive to higher-dimensional gravity scales in the range of 3 to 5 TeV, when the number of transverse dimensions varies from $p = 6$ at the subfermi to $p = 2$ at the submm, where the effect becomes stronger^{18,20}. This is comparable to the sensitivity on possible TeV dimensions with gauge interactions, as discussed previously. The most exciting possibility is of course when the available energy is bigger than the gravity scale, in which case gravitational interactions are strong and particle colliders become the best tools for studying quantum gravity.

When the string scale is of order of a few TeV, the question of gauge hierarchy is to understand why some dimensions transverse to our brane-world are so large or/and why the string coupling is so small. Thus, the large number to explain varies from 10^{15} up to 10^5 in the case of six transverse dimensions. This problem has a difficult part which is its dynamical aspect and a *technical* part that consists to guarantee its stability under quantum corrections. Supersymmetry solves this technical aspect by allowing only logarithmic dependence on the UV cutoff—usually taken at M_P —to the masses and other parameters of the Standard Model. Although naively one would expect that low string scale would also solve the problem by nullification¹⁴, it turns out that this is not true in general²¹. The reason is due to the appearance of local tadpoles in various processes on the world-brane, associated to the emission of a massless closed string in the transverse space. These tadpoles diverge linearly when the closed string propagates “effectively” in one preferred large transverse direction ($d_\perp = 1$) and logarithmically when $d_\perp = 2$. Local tadpole cancellation implies severe constraints for model building while their presence implies a set of conditions for the compactification manifold that forbids linear divergences. The remaining divergences are at most logarithmic and the technical aspect of gauge hierarchy is solved at the same level as with supersymmetry. Moreover, the case of $d_\perp = 2$ is singled out as the only one in which the origin of the hierarchy would not be attributed to ‘out of this world’ bulk physics, since the presence of logarithmic sensitivity could allow for a dynamical determination of the hierarchy by minimizing the effective potential.^b

Although the initial motivation of low energy supersymmetry is in principle lost, there may be still a reason to be introduced in the bulk, in connection to the cosmological constant and stability problem. In fact, the bulk energy density of a generic non-supersymmetric string model is $V_{\text{bulk}} \sim M_{\text{str}}^{4+p}$, giving rise to an energy density on our world-brane³:

$$V_{\text{brane}} \simeq V_{\text{bulk}} R_\perp^p \sim M_{\text{str}}^2 M_P^2, \quad (9)$$

by virtue of eqs. (7) and (8). This is analogous to the quadratically divergent contribution to the vacuum energy in softly broken supersymmetry, which in general destabilizes the hierarchy since it induces a large potential for R_\perp . On the other hand, its absence is guaranteed if the bulk remains approximately supersymmetric, which is the case when for instance supersymmetry is broken spontaneously primordially only on the brane (even maximally)¹⁵.

^b A similar conclusion appears to hold in low-scale type II models with infinitesimal coupling⁸.

3 Gravity modification at short distances

TeV scale strings with extra large transverse dimensions predict important modifications of gravitation at short distances that could be measured in experiments which test gravity in the submillimeter range (see fig. 1). These “tabletop” experiments, that cost orders of magnitude less than high energy particle accelerators, might provide an independent verification of the above ideas much before LHC. In fact, there are two classes of general predictions:

(i) Deviations from the Newton’s law $1/r^2$ behavior to $1/r^{2+p}$ which can be observable for $p = 2$ transverse dimensions of submm size. This case is particularly favored for theoretical reasons because of the logarithmic sensitivity of Standard Model couplings on the size of transverse space, as we discussed above, but also for phenomenological reasons, since the effects to particle colliders are maximally enhanced. Notice also the coincidence of this scale with the possible value of the cosmological constant in the universe that recent observations seem to support.

(ii) New scalar forces in the submm range mediated by light bulk scalar fields with masses M_{str}^2/M_P , motivated by the problem of supersymmetry breaking. Such a universal scalar corresponds to the transverse radius modulus $\phi_\perp \equiv \ln R_\perp$ in Planck units. Following our previous discussion, if supersymmetry is broken primordially only on the brane, its transmission to the bulk is gravitationally suppressed and ϕ_\perp acquires a mass proportional to the gravitino mass, of the order of¹⁵

$$m_{\phi_\perp}^2 \sim G_{N(4+p)} \frac{V_{\text{brane}}}{R_\perp^p} \simeq \frac{M_{\text{str}}^4}{M_P^2}, \quad (10)$$

using eq. (7). Unlike to the case of the TeV radius modulus ϕ (4), its coupling to matter is in general model dependent. Following the same steps as before, one finds that its coupling to nucleons relative to gravity α_{ϕ_\perp} is given by eq. (5) replacing R by R_\perp . It follows that α_{ϕ_\perp} is strongly suppressed, except again in the case of $d_\perp = 2$, when gauge couplings acquire logarithmic sensitivity on R_\perp . In this case, the resulting scalar force is of order $\mathcal{O}(1)$ relative to gravity –although not exactly computable– and therefore in principle within the expected sensitivity of present and future experiments displayed in fig. 1.

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